MATHEMATICS METHODS : UNITS 3 & 4, 2023
Thursday 17th August (T3W5)
Test 4 – Logarithmic Functions, Continuous Random Variables and the Normal Distribution (10%)
4.1.4, 4.1.6 – 4.1.14, 4.2.1 – 4.2.7, 4.3.1

Time Allowed 20 minutes	First Name	Surname	Marks / 23 marks

Circle your Teacher's Name:

Mrs Alvaro	Ms Chua	Mrs Fraser-Jones	Mrs Greenaway
Mr Luzuk	Mrs Murray	Ms Narendranathan	Mr Tanday

Assessment Conditions: *(N.B. Sufficient working out must be shown to gain full marks)*

❖ Calculators: Not Allowed

❖ Formula Sheet: Provided

❖ Notes: Not Allowed

PART A – CALCULATOR FREE

Question 1   (4 marks)

Determine the expected value and standard deviation for the continuous random variable X with the probability density function $f(x) = 6x(1 - x)$, for $0 \leq x \leq 1$.

Question 2   (6 marks = 2, 2, 2)

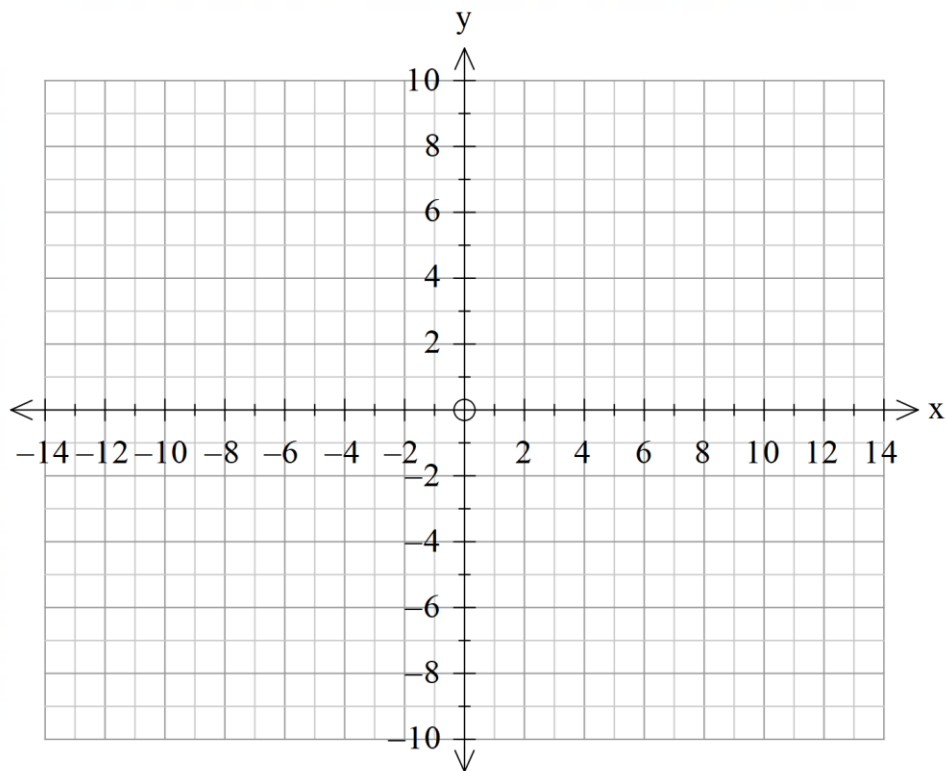Determine the following, expressing your answer in simplest form.

a) $\frac{dy}{dx}$ where $y = 2x^3 \ln(5x - 7)$

b) $\frac{dy}{dx}$ where $y = \ln\left(\frac{3x+1}{(x^2+3)^5}\right)$

c) $\int \frac{x^2-5x}{2x^3-15x^2-1}\,dx$

Question 3   (3 marks)

On the axes below, sketch the graph of $y = 3 - \log_2(4 + x)$, showing all key features.

Question 4   (4 marks = 2, 2)

A continuous random variable, X, has probability density function $f(x) = 2 \cos 2x$, for domain $0 \le x \le \frac{\pi}{4}$.
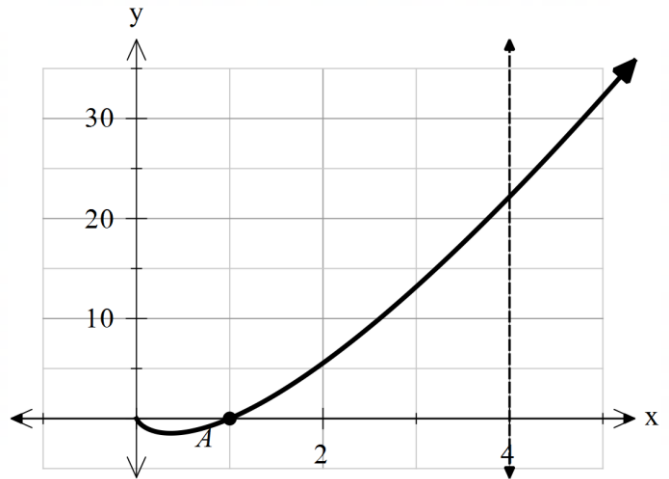
a) Demonstrate why $f(x)$ is suitable to be used as a probability density function.

b) Determine $P\left(X \ge \frac{\pi}{6}\right)$.

Question 5   (6 marks = 2, 4)

The graph of $y = 4x \ln x$ is shown below and point A is located at $\left(\frac{1}{\sqrt{e}}, 0\right)$.

a) Show that $\frac{d}{dx}(x^2 \ln x) = 2x \ln x + x$

b) Hence, determine the exact area enclosed by the graph of $y = 4x \ln x$, the x-axis and $x = 4$.

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Time Allowed	First Name	Surname	Marks
30 minutes			/ **30** marks

Circle your Teacher's Name:

Mrs Alvaro	Ms Chua	Mrs Fraser-Jones	Mrs Greenaway
Mr Luzuk	Mrs Murray	Ms Narendranathan	Mr Tanday

Assessment Conditions: *(N.B. Sufficient working out must be shown to gain full marks)*

❖ Calculators: Allowed

❖ Formula Sheet: Provided

❖ Notes: Not Allowed

PART B – CALCULATOR ASSUMED

Question 6 **(6 marks = 1, 3, 2)**

The continuous random variable X has probability density function $f(x) = \dfrac{3x^2(4-x)}{64}$, $0 \leq x \leq 4$.

Determine each of the following.

a) $P(X > 3)$

b) The probability of being within one standard deviation of the mean.

c) The median of X.

Question 7   (3 marks)

A logarithmic scale with base 5 is used and the distance measured on the axis between the notches representing 10 and 15 is 2.02 cm. What is the distance, correct to one decimal place, between the notches for 5 and 125?

2.02 cm

5   10   15   125   *Diagram is not drawn to scale.*
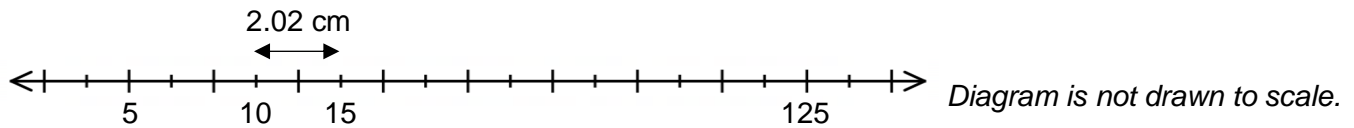
Question 8   (4 marks = 1, 3)

The lifetime, T, of a new battery used for a scientific calculator is a continuous random variable where the probability that it will run flat t weeks after it is first inserted is modelled with the probability density function

$$f(t) = \frac{e^{-0.25t}}{4} \text{ for } t \geq 0.$$

a) Determine the probability that the battery will last at least 6 weeks.

b) A cautious teacher buys 10 spare scientific calculators at the beginning of the term and makes sure they all have a new battery inserted. Determine the probability that more than half of the calculators will last at least 6 weeks.

Question 9   (4 marks = 2, 2)
The graphs of the probability density functions for four different normal distributions, A, B, C and D, are shown below.
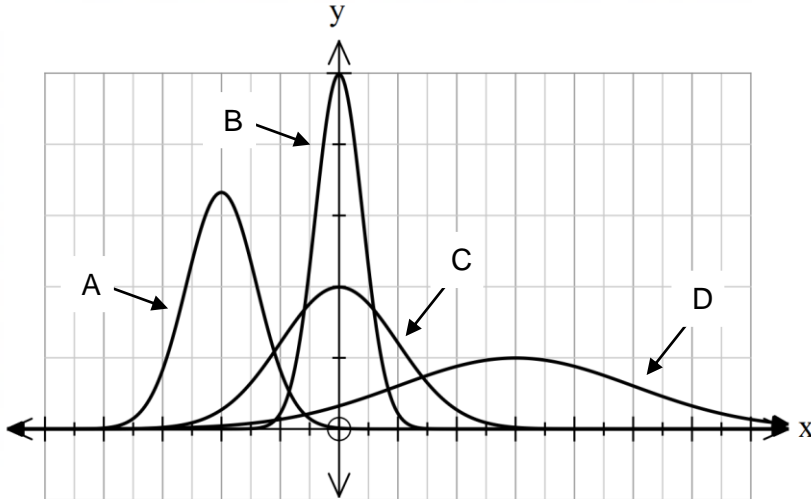
a) Which of the four distributions has the largest mean? Justify your answer.

b) State the four distributions in <u>ascending</u> order of their standard deviations. Justify your answer.

Question 10   (3 marks)
Students learning to use a woodworking lathe take an average time of 35 minutes to make a simple figure. 30% of the students could finish their figure within 30 minutes. If the times, T, are normally distributed, determine the probability that a student took more than 38 minutes to make their figure.

Question 11   (6 marks = 2, 1, 3)

The continuous random variable X is normally distributed with mean 18 and variance 5.

a) Determine the following probabilities:

   (i) $P(12 < X \leq 17)$

   (ii) $P(X > 20 \mid X < 24)$

b) What is the third quartile of X?

c) A second random variable, Y, is normally distributed with mean -1.6 and variance 7.2. Y can be expressed as a linear transformation of X, i.e. $Y = aX + b$. Determine the values of a and b.

Question 12   (4 marks = 1, 3)

The weight of a regulation baseball is required to be between 142 grams and 149 grams.

a) The supplier, Rawlings, of baseballs for the national league has found that the weight of their manufactured baseballs, W, is normally distributed with a mean of 145.5 grams and standard deviation 1.8 grams. Determine the probability that one of their baseballs will not meet the regulation requirements.

b) A startup company, Crawlings, is testing their new machinery and find that the weight of their baseballs, C, have a 10.6% chance of being too light, and 4.8% chance of being too heavy. What is the mean and standard deviation for the weight of their baseballs?